Classical Mechanics.
An intuitive approach, based on the pursuit of deeper understanding through the use of simple familiar thought experiments.

We can imagine that to be where we currently stand at the current level of scientific knowledge that perhaps average people in the old days had to start by tackling the simple (in retrospect) problems first, those problems which were more familiar to them at the time.
We can then hypothesize that it was through their individual efforts for solving these “simple” problems that the advancement of scientific knowledge was possible; a true statement. But one must not forget how they came to solve these problems in the first place for it is in the story that lies their reasoning and a solid understanding of their results (formulas, theories, equations ...etc). It is however common practice to forgo the study of their reasoning and encourage the utilization of their formulas for that is all is needed for us to make use of current technology. We should digress from these standards and pursue true knowledge, the reward is the pleasure of discovery and the development of new exciting technologies for generations to come.

In this lecture we will cover simple concepts of everyday use i.e. position, velocity and acceleration. As a simple example, it might be fruitful to expand on a potential story or thought experiment to provide some context as we go along.

As a start we might hypothesize that the prime necessity of people in ancient times was locating objects around their environment (where is my sack of coins? My wife?) and as such a coordinate system for locating objects based on their current position (potentially the origin) was envisioned. For example, we could say, my sack of coins is 3 steps to my right and one step forward. Or mathematically we could write:

$$s = 3(\text{right}) + 1(\text{forward}) = 3x + 1y$$

That was all great but for larger distances people might have been interested in recording their progress towards reaching a desired location. e.g. an army officer might say, I normally travel from Naples (location s) to Salerno (location s'); covering a distance (let's use meters for simplicity) between the two cities of 40 km (sometimes expressed as $\Delta s = 40$ km, where Δ means “change in”) in 1 day, therefore having an approximate measure of how quickly he and his army could move (40 km per day), later this measure of progress with time was referred to as “speed” (or velocity if we take into account the direction of the displacement), and based on this information the officer could make better decisions as we shall see shortly.

Knowing his army speed (40km/day), the officer decides he wants to know how long it would take him to pursue an enemy army 60 km away advancing at 20km/day.

The officer knows he is faster than the enemy army by the difference of their speeds or 40-20 = 20 km/day:

$$v\_diff = 20 \text{ km /day}$$

so he realizes that it will take him 60km / (20 km a day) or simply 3 days to reach the advancing enemy. Eagerly and having understood speed as a simple measure of progress, he makes the necessary arrangements and embarks on the pursuit to reach the enemy.

The army proceeds with the advance and after 5 days finally the enemy is at sight and the attack takes place at the start of the 6^{th} day; most of the enemy troops are defeated except for the enemy captain which flees the battlefield at an unreachable rate of 60 km/day.

While the attack was victorious the officer has a sour after taste from this experience; more man were lost than he had anticipated. This was due to the pursuit taking longer than expected which caused troops fatigue, food supplies to start running short and a lower than optimal morale.

Throughout the trip back home, he is haunted to figure out exactly what went wrong. Fortunately he recorded his progress during the pursuit as follows:

day1: 40 km (speed: 40km / day);

day2: 35 km (speed: 35km / day)

day3: 30 km (speed: 30km / day)

day4: 30 km (speed: 30km / day)

day5: 25 km (speed: 25km / day)

Here lies the clue to the problem; the speed of the troops was not constant every day but changed from day to day. Over the 5 days the troops traveled:

$$s\_total = 40 \text{ km} + 35 \text{ km} + 30 \text{ km} + 25 \text{ km} = 160 \text{ km}$$

with an average speed of:

$$v\ \_avg = 160 \text{ km} / 5 \text{ days} = 32 \text{ km} / \text{day}$$

this explains why it took them 5 days to reach the enemy; the difference in their speeds was really 32 – 20 = 12 km / day:

$$v\_diff1 = 12 \text{ km} / \text{day}$$

therefore to reach the target it took 60 km / (12 km a day) or 5 days, which is consistent with the officer results.

Having settled this debate, the officer realizes that to truly have an idea of the capability of his troops he has to take into account the impact that fatigue has as the days go along. In other words by how much is the speed of the troops changing (Δv) per every new day of pursuit ($\Delta t = 1$ day). This measure of speeding up or slowing down is called acceleration (a); That is what we want to calculate:

$$a = \Delta v / \Delta t = \Delta v / \text{day}$$

So let's gather the data we need first:

day1: 40 km (speed: 40km / day)

$\Delta v = 35 – 40 = -5$ km/day $a = (-5 \text{ km} / \text{day})/\text{day}$

day2: 35 km (speed: 35km / day)

$\Delta v = 30 – 35 = -5$ km/day $a = (-5 \text{ km} / \text{day})/\text{day}$

day3: 30 km (speed: 30km / day)

$\Delta v = 30 – 30 = 0$ km/day　　　　a = (0 km / day)/day

day4: 30 km (speed: 30km / day)

$\Delta v = 25 – 30 = -5$ km/day　　　　a = (-5 km / day)/day

day5: 25 km (speed: 25km / day)

As we can see fatigue is evident in the troops, they often seem to be slowing down for every day of pursuit; in average they are slowing down by 3.75 km/day with every day that passes by or mathematically:

$$a_{avg}=\frac{\frac{-5\,km}{day}+\frac{-5\,km}{day}+\frac{0\,km}{day}+\frac{-5\,km}{day}}{4*day}=\frac{\frac{-3.75\,km}{day}}{day}=\frac{-3.75\,km}{day^2}$$

The average acceleration in this case is analogous to the rate of fatigue of his army and is a very useful practical piece of information.

Consider the following case, the officer is asked to pursue an enemy army traveling at 30 km /day, with the enemy army again located 60 km away.

We know his army's initial speed (v_a1) and the enemy's army speed (v_a2):

v_a1 = 40 km/day　　　　v_a2 = 30 km/day

and their difference:

v_diff = 40 – 30 km / day = 10 km /day

without further study the officer could wrongly assume he would reach the enemy army at the end of the 6th day, however considering the rate of fatigue of his army, we can see that the pursuit campaign is actually futile.

day1: speed: 40km / day	v_diff = 10 km/day
day2: speed: 40 km /day – 3.75 km /day = 36.25	v_diff = 6.25 km/day
day3: speed: 36.25 km /day – 3.75 km /day = 32.5	v _diff = 2.5 km /day
day4: speed: 32.5 km /day – 3.75 km /day = 28.75	v_diff = -1.25 km /day

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On the 4th day of traveling his army has slowed down to a speed below that of the enemy army and they are starting to loose ground, at this pace their best bet would be to turn back and retreat home. We can all agree that:

“Understanding science even in its simplest forms can help us make better more informed decisions”

Something that we have not consider so far is what happens as we let the time scale get smaller and smaller e.g. instead of displacement per day, we could consider displacement per hour or perhaps per second and keep shrinking the scale on and on ad infinitum. However consider the following situation:

You find yourself speeding up and the police pulls you over (let's say things develop in a friendly manner, and the policeman has some background in mathematics); the following conversation develops:

Police: I am afraid you were driving over the speed limit at 70 km / hour in a speed zone restricted to 50 km / hour.
You: wait sir, I have only been driving for 10 minutes, clearly your time scale is not accurate.
Police: Well this is simply a matter of algebra 70 km / hour is the same as 1.167 km / minute.
You: OK but how can you tell what I was doing a minute ago; for all you know I could have been speeding up in the beginning and slowing down in the end; my average velocity being 1.67 km / min but clearly at the time you took the measurement my velocity should have been lower since I was actually slowing down.
Police: This is good reasoning, we could keep getting the timescale smaller and smaller, however to prove to you that you were traveling above the speed limit I would have to tell you what your speed was exactly at the instant you crossed the radar.
You: So I proved my point, no speed ticket then eh?
Police: No, unfortunately for you we can use mathematics to settle this argument; we can use a nifty tool called Differential Calculus to calculate your instantaneous speed i.e. the distance traveled in an instant of time.
In fact by measuring your position at different intervals we can gather an approximate mathematical description of your movement (displacement function), taking the differential of the displacement function with respect to time (letting the timescale approach 0.) we can find the velocity function, then we can calculate what your speed was at the instant you passed the radar; *here is your ticket*.

Calculus is a very useful and intuitive tool, a subject of other lectures, *you should take some time to learn it.*

Our equations using calculus have a similar form as before, mainly:

Instantaneous Velocity:

$$v=\frac{\Delta s}{\Delta t} \rightarrow \frac{ds}{dt}$$

This is read as: The instantaneous velocity is the change in displacement per time as we let our time scale approach 0.

Instantaneous Acceleration:

$$a=\frac{\Delta v}{\Delta t} \rightarrow \frac{dv}{dt}$$

this is read as: The instantaneous acceleration is the change in Velocity per time as we let our time scale approach 0.